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Symmetric Functions of the 13^{ic}.

BY CAPTAIN P. A. MACMAHON, R. A.

The following table represents the symmetric functions of the roots of an equation of the 13th degree, arranged according to Mr. Durfee's plan.

In addition to Professor Cayley's law of symmetry, the following method was carried out in order to ensure the correctness of the numbers: viz. the equation being $x^n - a_1 x^{n-1} + a_2 x^{n-2} - \dots + (-)^n a_n = 0$, and any product of coefficients

$$a_\lambda^l a_\mu^m a_\nu^n \dots ;$$

if the $(r+1)$ -agonal weight of the term be defined to be

$$w_{r+1} = l(r+1)_\lambda - r+1 + m(r+1)_\mu - r+1 + n(r+1)_\nu - r+1 + \dots ,$$

wherein $(r+1)_t$ denotes the t^{th} of the $(r+1)$ -agonal numbers, then I have elsewhere shown (*vide* Proc. Lond. Math. Soc.) that denoting the sum of all those symmetric functions of weight w ($w \nless n$), which contain r parts in their partitions, by S_w^r ,

$$S_w^r = \Sigma (-)^{w+k+r-1} \frac{(k-1)! w_{r+1}}{l! m! r!} a_\lambda^l a_\mu^m a_\nu^n \dots ,$$

where

$$w = l\lambda + m\mu + n\nu + \dots ,$$

$$k = l + m + n + \dots .$$

From this we can, by the law of symmetry, immediately write down the value of any symmetric function of the equation

$$x^n - cx^{n-1} + cx^{n-2} - cx^{n-3} + \dots + (-)^n c = 0 ;$$

thus, turning to the partition notation,

$$(\lambda^l . \mu^m . \nu^n \dots) = \sum_{r=\lambda}^{r=1} (-)^{w_2+k+r-1} \frac{(k-1)! w_{r+1}}{l! m! n!} c^r .$$

w_2 , w_{r+1} and k referring to the symmetric function.

In the annexed table, the coefficients were supposed to be each c , and the results compared with those obtained from the above formula.

As a further check, advantage was taken of the fact that every symmetric function (except those whose partitions are composed wholly of units) of the equation

$$x^n - \frac{x^{n-1}}{1!} + \frac{x^{n-2}}{2!} - \frac{x^{n-3}}{3!} + \dots + (-)^n \frac{1}{n!} = 0,$$

vanishes; the weight of course being ∇n .

Consequently we must obtain zero, on substituting for each coefficient the reciprocal of the factorial of its suffix.

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	(13)	(12.1)	(11.2)	(10.3)	(9.4)	(8.5)	(7.6)	(11.1 ²)	(10.2.1)	(9.3.1)	(9.2 ²)	(8.4.1)	(8.3.2)	(7.5.1)	(7.4.2)	(7.3 ²)	(6 ² .1)	(6.5.2)	(6.4.3)	(5 ² .3)
(13)	+13	-13	-13	-13	-13	-13	-13	+13	+26	+26	+13	+26	+26	+26	+26	+13	+13	+26	+26	+13
(12.1)	-13	+1	+13	+13	+13	+13	+13	-1	-14	-14	-13	-14	-26	-14	-26	-13	-7	-26	-26	-13
(11.2)	-13	+13	-9	+13	+13	+13	+13	-2	-4	-26	+9	-26	-4	-26	-4	-13	-13	-4	-26	-13
(10.3)	-13	+13	+13	-17	+13	+13	+13	-13	+4	+4	-13	-26	+4	-26	-26	+17	-13	-26	+4	+2
(9.4)	-13	+13	+13	+13	-23	+13	+13	-13	-26	+10	+5	+10	-26	-26	+10	-13	-13	-26	+10	-13
(8.5)	-13	+13	+13	+13	+13	-27	+13	-13	-26	-26	-13	+14	+14	+14	-26	-13	-13	+14	-26	+27
(7.6)	-13	+13	+13	+13	+13	+13	-29	-13	-26	-26	-13	-26	-26	+16	+16	+8	+29	+16	+16	-13
(11.1 ²)	+13	-1	-2	-13	-13	-13	-13	+1	+3	+14	+2	+14	+15	+14	+15	+13	+7	+15	+26	+13
(10.2.1)	+26	-14	-4	+4	-26	-26	-26	+3	+8	+10	+4	+40	+0	+40	+30	-4	+20	+30	+22	+11
(9.3.1)	+26	-14	-26	+4	+10	-26	-26	+14	+10	+1	+8	+4	+22	+40	+16	-4	+20	+52	-14	+11
(9.2 ²)	+13	-13	+9	-13	+5	-13	-13	+2	+4	+8	0	+8	+4	+26	-14	+13	+13	+4	+8	+13
(8.4.1)	+26	-14	-26	-26	+10	+14	-26	+14	+40	+4	+8	-4	+12	0	+16	+26	+20	+12	+16	-14
(8.3.2)	+26	-26	-4	+4	-26	+14	-26	+15	+0	+22	+4	+12	+8	+12	+30	-4	+26	-10	+22	-29
(7.5.1)	+26	-14	-26	-26	-26	+14	+16	+14	+40	+40	+26	0	+12	-7	+10	+5	-22	-30	+10	-14
(7.4.2)	+26	-26	-4	-26	+10	-26	+16	+15	+30	+16	-14	+16	+30	+10	+8	+5	-16	-12	-26	+26
(7.3 ²)	+13	-13	-13	+17	-13	-13	+8	+13	-4	-4	+13	+26	-4	+5	+5	+4	-8	+5	-25	-2
(6 ² .1)	+13	-7	-13	-13	-13	-13	+29	+7	+20	+20	+13	+20	+26	-22	-16	-8	-14	-16	-16	+13
(6.5.2)	+26	-26	-4	-26	-26	+14	+16	+15	+30	+52	+4	+12	-10	-30	-12	+5	-16	+8	+10	-14
(6.4.3)	+26	-26	-26	+4	+10	-26	+16	+26	+22	-14	+8	+16	+22	+10	-26	-25	-16	+10	+16	+11
(5 ² .3)	+13	-13	-13	+2	-13	+27	-13	+13	+11	+11	+13	-14	-29	-14	+26	-2	+13	-14	+11	+3
(5.4 ²)	+13	-13	-13	-13	+23	+7	-13	+13	+26	-10	-5	-30	+6	+6	-10	+13	+13	+6	-10	-7
(10.1 ³)	-13	+1	+2	+3	+13	+13	+13	-1	-3	-4	-2	-14	-5	-14	-15	-3	-7	-15	-16	-8
(9.2.1 ²)	-39	+15	+6	+9	+3	+39	+39	-4	-11	-15	-6	-18	-15	-54	-9	-9	-27	-45	-12	-24
(8.3.1 ²)	-39	+15	+28	+9	+3	-1	+39	-15	-13	-15	-10	-10	-21	-14	-31	-9	-27	-27	-12	+16
(8.2 ² .1)	-39	+27	-5	+9	+21	-1	+39	-5	-12	-18	-4	-24	-12	-26	-16	-9	-33	+6	-30	+16
(7.4.1 ²)	-39	+15	+28	+39	+3	-1	-3	-15	-43	-18	-10	-10	-27	-7	-17	-18	+15	+15	0	+1
(7.3.2.1)	-78	+54	+34	-12	+42	+88	-6	-32	-18	-33	-16	-56	-30	-43	-48	-9	+18	+12	+54	+7
(7.2 ³)	-13	+13	-9	+13	-5	+13	-1	-2	-4	-8	0	-8	-4	-12	0	-6	+1	+10	+6	-13
(6.5.1 ²)	-39	+15	+28	+39	+39	-1	-45	-15	-43	-54	-28	-14	-27	+35	+17	+3	+21	+27	+6	+1
(6.4.2.1)	-78	+54	+34	+48	+6	+88	-48	-32	-78	-30	+2	-52	-42	+34	+30	+15	+24	-6	0	-23
(6.3 ² .1)	-39	+27	+39	-21	+3	+39	-24	-27	-6	+3	-21	-30	-18	-3	+21	+21	+12	-15	+9	-9
(6.3.2 ²)	-39	+39	-5	+9	+21	-1	-3	-17	-4	-30	-4	-20	-12	+4	+26	+12	+3	-12	-24	+16
(5 ² .2.1)	-39	+27	+17	+24	+39	-41	-3	-16	-39	-51	-17	+14	+39	+21	-14	-3	+9	+6	-21	+11

	(5.4 ²)	(10.1 ³)	(9.2.1 ²)	(8.3.1 ²)	(8.2 ² .1)	(7.4.1 ²)	(7.3.2.1)	(7.2 ³)	(6.5.1 ²)	(6.4.2.1)	(6.3 ² .1)	(6.3.2 ²)	(5 ² .2.1)	(5.4.3.1)	(5.4.2 ²)	(5.3 ² .2)	(4 ³ .1)	(9.1 ⁴)	(8.2.1 ³)	(7.3.1 ³)
(13)	+13	-13	-39	-39	-39	-39	-78	-13	-39	-78	-39	-39	-39	-78	-39	-39	-13	+13	+52	+52
(12.1)	-13	+1	+15	+15	+27	+15	+54	+13	+15	+54	+27	+39	+27	+54	+39	+39	+9	-1	-16	-16
(11.2)	-13	+2	+6	+28	-5	+28	+34	-9	+28	+34	+39	-5	+17	+78	-5	+17	+13	-2	-8	-30
(10.3)	-13	+3	+9	+9	+9	+39	-12	+13	+39	+48	-21	+9	+24	+18	+39	-21	+13	-3	-12	-12
(9.4)	+23	+13	+3	+3	+21	+3	+42	-5	+39	+6	+3	+21	+39	-30	-15	+39	-23	-4	-16	-16
(8.5)	+7	+13	+39	-1	-1	-1	+38	+13	-1	+38	+39	-1	-41	-42	-1	-41	-7	-13	-12	-12
(7.6)	-13	+13	+39	+39	+39	-3	-6	-1	-45	-48	-24	-3	-3	+36	-3	+18	+13	-13	-52	-10
(11.1 ²)	+13	-1	-4	-15	-5	-15	-32	-2	-15	-32	-27	-17	-16	-54	-17	-28	-9	+1	+5	+16
(10.2.1)	+26	-3	-11	-13	-12	-43	-18	-4	-43	-78	-6	-4	-39	-72	-34	+4	-22	+3	+14	+16
(9.3.1)	-10	-4	-15	-15	-18	-18	-33	-8	-54	-30	+3	-30	-51	+9	-24	-18	+14	+4	+19	+19
(9.2 ²)	-5	-2	-6	-10	-4	-10	-16	0	-28	+2	-21	-4	-17	-24	+14	-17	+5	+2	+8	+12
(8.4.1)	-30	-14	-18	-10	-24	-10	-56	-8	-14	-52	-30	-20	+14	+64	+16	+2	+18	+5	+24	+24
(8.3.2)	+6	-5	-15	-21	-12	-27	-30	-4	-27	-42	-18	-12	+39	+24	+6	+36	-6	+5	+20	+26
(7.5.1)	+6	-14	-54	-14	-26	-7	-43	-12	+35	+34	-3	+4	+21	-5	+4	+23	-2	+14	+28	+21
(7.4.2)	-10	-15	-9	-31	-16	-17	-48	0	+17	+30	+21	+26	-14	-6	+6	-35	+10	+6	+24	+32
(7.3 ²)	+13	-3	-9	-9	-9	-18	-9	-6	+3	+15	+21	+12	-3	+3	-18	0	-13	+3	+12	+12
(6 ² .1)	+13	-7	-27	-27	-33	+15	+18	+1	+21	+24	+12	+3	+9	-24	+3	-18	-11	+7	+34	-8
(6.5.2)	+6	-15	-45	-27	+6	+15	+12	+10	+27	-6	-15	-12	+6	+6	-12	+15	-6	+15	+20	0
(6.4.3)	-10	-16	-12	-12	-30	0	+54	+6	+6	0	+9	-24	-21	-18	+18	+3	+10	+7	+28	-14
(5 ² .3)	-7	-8	-24	+16	+16	+1	+7	-13	+1	-23	-9	+16	+11	-3	+1	-4	+7	+8	-8	-8
(5.4 ²)	+7	-13	-3	+17	-1	+17	-22	+5	-19	+14	-3	-1	+1	+10	-5	+1	-7	+4	-4	-4
(10.1 ³)	-13	+1	+4	+5	+5	+15	+12	+2	+15	+32	+7	+7	+16	+34	+17	+8	+9	-1	-5	-6
(9.2.1 ²)	-3	+4	+15	+19	+17	+22	+41	+6	+58	+38	+24	+21	+55	+45	+15	+24	-5	-4	-19	-23
(8.3.1 ²)	+17	+5	+19	+22	+23	+25	+49	+10	+29	+54	+24	+31	-13	-43	+1	-10	-9	-5	-24	-27
(8.2 ² .1)	-1	+5	+17	+23	+16	+29	+42	+4	+31	+52	+27	+16	-24	-8	-20	-19	+5	-5	-22	-28
(7.4.1 ²)	+17	+15	+22	+25	+29	+18	+67	+10	-20	-14	-6	-5	-5	-17	-13	+5	-9	-6	-29	-33
(7.3.2.1)	-22	+12	+41	+49	+42	+67	+88	+16	-9	-46	-42	-38	+2	-10	+24	-1	+14	-12	-53	-61
(7.2 ³)	+5	+2	+6	+10	+4	+10	+16	0	0	-16	0	-10	+3	+10	0	+10	-5	-2	-8	-12
(6.5.1 ²)	-19	+15	+58	+29	+31	-20	-9	0	-32	-26	+3	+1	-17	+17	+1	-1	+11	-15	-33	+5
(6.4.2.1)	+14	+32	+38	+54	+52	-14	-46	-16	-26	+4	-3	+10	+15	+2	-12	+17	-6	-14	-62	+12
(6.3 ² .1)	-3	+7	+24	+24	+27	-6	-42	0	+3	-3	-24	+12	+12	+18	0	-3	-1	-7	-31	+11
(6.3.2 ²)	-1	+7	+21	+31	+16	-5	-38	-10	+1	+10	+12	+16	-4	-6	-2	-10	+1	-7	-28	+4
(5 ² .2.1)	+1	+16	+55	-13	-24	-5	+2	+3	-17	+15	+12	-4	-14	+8	+11	-11	-5	-16	+9	+4

	(7.2 ² .1 ²)	(6.4.1 ³)	(6.3.2.1 ²)	(6.3 ³ .1)	(5 ² .1 ³)	(5.4.2.1 ²)	(8.1 ⁵)	(7.2.1 ⁴)	(6.3.1 ⁴)	(7.1 ⁶)	(5.3 ² .1 ²)	(4 ² .3.2)	(6.2 ² .1 ³)	(6.2.1 ⁵)	(6.1 ⁷)	(5.3.2 ² .1)	(5.2 ⁴)	(5.4.1 ⁴)	(5.3.2.1 ³)	(5.3 ³ .1 ²)
(13)	+78	+52	+156	+52	+26	+156	-13	-65	-65	+13	+78	-39	-180	+78	-13	+156	+13	-65	-260	-130
(12.1)	-42	-16	-84	-40	-8	-84	+1	+17	+17	-1	-42	+39	+58	-18	+1	-120	-13	+17	+116	+82
(11.2)	-1	-30	-68	+14	-15	-68	+2	+10	+32	-2	-67	+17	+9	-12	+2	-24	+9	+32	+106	-13
(10.3)	-18	-42	-6	-22	-21	-96	+3	+15	+15	-3	+12	+9	+30	-18	+3	-6	-13	+45	+30	+40
(9.4)	-24	-16	-48	-16	-26	-12	+4	+20	+20	-4	-6	-33	+40	-24	+4	-84	+5	+20	+80	+40
(8.5)	-38	-12	-76	-12	+14	+44	+5	+25	+25	-5	+22	+19	+50	-30	+5	+84	-3	-15	-60	-30
(7.6)	-15	+32	+54	+4	+16	+12	+13	+23	-19	-6	-15	-3	-17	+6	0	-30	+1	-19	+8	+11
(11.1 ²)	+9	+16	+51	+7	+8	+51	-1	-6	-17	+1	+42	-28	-14	+7	-1	+54	+2	-17	-72	-16
(10.2.1)	+23	+46	+42	+16	+23	+132	-3	-17	-19	+3	+19	-26	-37	+20	-3	+34	+4	-49	-72	-39
(9.3.1)	+33	+22	+63	+26	+29	+63	-4	-23	-23	+4	+12	+24	-52	+27	-4	+81	+8	-26	-101	-59
(9.2 ²)	+10	+12	+32	+4	+15	+14	-2	-10	-14	+2	+31	+19	-18	+12	-2	+24	0	-14	-52	-14
(8.4.1)	+42	+24	+84	+32	-6	-40	-5	-29	-29	+5	-20	+14	-66	+34	-5	-20	-2	+11	+28	+6
(8.3.2)	+27	+32	+66	+16	-4	-36	-5	-25	-31	+5	-21	-30	-47	+30	-5	-66	-6	+3	+48	+37
(7.5.1)	+52	-21	-15	-4	-13	-23	-6	-35	+7	+6	-8	-16	+4	-1	0	-20	+2	+12	+21	-3
(7.4.2)	+32	-10	-38	-26	-1	-4	-6	-30	+4	+6	+10	+2	+28	-6	0	+38	0	+4	-18	-6
(7.3 ²)	+18	0	-36	-6	0	+12	-3	-15	+6	+3	-12	+12	+12	-3	0	+6	+6	-3	+12	-12
(6 ² .1)	-3	-14	-18	+2	-7	-12	-7	+1	+7	0	+15	+3	-1	0	0	+12	-1	+7	-8	+1
(6.5.2)	-24	-12	+24	+2	-11	+6	-7	+7	-3	0	-3	+6	-2	0	0	-6	0	+7	-4	+2
(6.4.3)	-21	-2	-12	+18	+5	+12	-7	+7	+7	0	+3	-6	-7	0	0	0	-6	-5	-2	+3
(5 ² .3)	+8	+7	+1	-3	-4	+1	0	0	0	0	+8	-4	0	0	0	-9	+3	0	0	0
(5.4 ²)	+4	-4	+8	-4	+6	-8	0	0	0	0	-4	+3	0	0	0	+4	0	0	0	0
(10.1 ³)	-9	-16	-21	-7	-8	-51	+1	+6	+7	-1	-12	+18	+14	-7	+1	-24	-2	+17	+32	+16
(9.2.1 ²)	-32	-26	-75	-23	-31	-75	+4	+23	+27	-4	-43	-18	+51	-27	+4	-79	-6	+30	+117	+55
(8.3.1 ²)	-42	-30	-90	-33	+3	+22	+5	+29	+32	-5	+14	-4	+66	-34	+5	+17	0	-5	-19	-5
(8.2 ² .1)	-33	-34	-82	-20	+2	+22	+5	+27	+33	-5	+10	+11	+55	-32	+5	+46	+6	-1	-28	-27
(7.4.1 ²)	-51	+9	+12	+3	+5	+24	+6	+35	-3	-6	+6	0	-4	+1	0	-6	0	-8	-5	+3
(7.3.2.1)	-83	+5	+74	+26	+1	-12	+12	+65	-11	-12	+18	+4	-32	+7	0	-7	-6	+2	-15	+12
(7.2 ³)	-10	+2	+10	+10	-1	0	+2	+10	0	-2	-10	-5	-10	+2	0	-10	0	0	+10	0
(6.5.1 ²)	+2	+17	0	-1	+11	+8	+7	-1	-2	0	-7	+2	+1	0	0	+2	0	-7	+3	-1
(6.4.2.1)	+29	+6	+2	-6	+2	+2	+14	-8	-4	0	-6	-2	+3	0	0	-16	+6	-2	+8	-3
(6.3 ² .1)	+12	+2	+21	-12	-5	-15	+7	-4	-7	0	0	-6	+4	0	0	+3	0	+5	-1	0
(6.3.2 ²)	+26	-2	-14	-6	+1	+4	+7	-7	+3	0	+2	+5	+2	0	0	+6	0	-1	-2	0
(5 ² .2.1)	-3	+1	-8	+3	+8	-10	0	0	0	0	-4	-2	0	0	0	+11	-3	0	0	0

	$(3^4.1)$	$(3^3.2^2)$	$(3^3.2.1^2)$	$(3^2.2^3.1)$	(3.2^5)	$(3^3.1^4)$	$(3^2.2^2.1^3)$	$(3.2^4.1^2)$	$(3^2.2.1^5)$	$(3.2^3.1^4)$	$(3^2.1^7)$	$(3.2^2.1^6)$	$(3.2.1^8)$	(3.1^{10})	$(2^6.1)$	$(2^5.1^3)$	$(2^4.1^5)$	$(2^3.1^7)$	$(2^2.1^9)$	$(2.1^{11}.1)$	(1^{13})
(13)	+13	+26	-130	-130	-13	+65	+390	+195	-273	-455	+52	+364	-117	+13	+13	-91	+182	-156	+65	-13	+1
(12.1)	-10	-26	+82	+106	+13	-25	-210	-135	+93	+215	-10	-112	+21	-1	-11	+55	-77	+44	-11	+1	
(11.2)	-13	-4	+75	-2	-9	-43	-104	+36	+97	+15	-19	-45	+18	-2	+9	-30	+27	-9	+1		
(10.3)	+17	+19	-80	-35	+7	+20	+105	0	-42	-35	+3	+21	-3	0	-7	+14	-7	+1			
(9.4)	-1	-20	+10	+40	-5	+10	-30	-15	-6	+20	+2	-4	0	0	+5	-5	+1				
(8.5)	-13	+14	+30	-30	+3	-5	-10	+15	+5	-5	0	0	0	0	-3	+1					
(7.6)	+8	-5	-17	+11	-1	-2	+9	-6	0	0	0	0	0	0	+1						
(11.1 ²)	+10	+15	-60	-40	-2	+25	+100	+25	-60	-50	+10	+35	-10	+1							
(10.2.1)	-7	-15	+33	+31	+2	-7	-46	-16	+15	+20	-1	-8	+1								
(9.3.1)	-10	+1	+28	-8	-2	-13	-12	+9	+12	-6	-2	+1									
(9.2 ²)	+7	+7	-21	-7	0	+7	+14	0	-7	0	+1										
(8.4.1)	+11	+6	-24	-2	+2	+3	+12	-4	-3	+1											
(8.3.2)	-4	-7	+9	+5	0	-1	-5	0	+1												
(7.5.1)	+2	-9	0	+8	-2	+2	-4	+1													
(7.4.2)	-7	+3	+6	-3	0	-2	+1														
(7.3 ²)	+4	+2	-4	0	0	+1															
(6 ² .1)	-5	+5	+5	-5	+1																
(6.5.2)	+5	-1	-3	+1																	
(6.4.3)	-1	-2	+1																		
(5 ² .3)	-2	+1																			
(5.4 ²)	+1																				

	(13)	(12.1)	(11.2)	(10.3)	(9.4)	(8.5)	(7.6)	(11.1 ²)	(10.2.1)	(9.3.1)	(9.2 ²)	(8.4.1)	(8.3.2)	(7.5.1)	(7.4.2)	(7.3 ²)	(6 ² .1)	(6.5.2)	(6.4.3)	(5 ² .3)
(5.4.3.1)	-78	+54	+78	+18	-30	-42	+36	-54	-72	+9	-24	+64	+24	-5	-6	+3	-24	+6	-18	-3
(5.4.2 ²)	-39	+39	-5	+39	-15	-1	-3	-17	-34	-24	+14	+16	+6	+4	+6	-18	+3	-12	+18	+1
(5.3 ² .2)	-39	+39	+17	-21	+39	-41	+18	-28	+4	-18	-17	+2	+36	+23	-35	0	-18	+15	+3	-4
(4 ³ .1)	-13	+9	+13	+13	-23	-7	+13	-9	-22	+14	+5	+18	-6	-2	+10	-13	-11	-6	+10	+7
(9.1 ⁴)	+13	-1	-2	-3	-4	-13	-13	+1	+3	+4	+2	+5	+5	+14	+6	+3	+7	+15	+7	+8
(8.2.1 ³)	+52	-16	-8	-12	-16	-12	-52	+5	+14	+19	+8	+24	+20	+28	+24	+12	+34	+20	+28	-8
(7.3.1 ³)	+52	-16	-30	-12	-16	-12	-10	+16	+16	+19	+12	+24	+26	+21	+32	+12	-8	0	-14	-8
(7.2 ² .1 ²)	+78	-42	-1	-18	-24	-38	-15	+9	+23	+33	+10	+42	+27	+52	+32	+18	-3	-24	-21	+8
(6.4.1 ³)	+52	-16	-30	-42	-16	-12	+32	+16	+46	+22	+12	+24	+32	-21	-10	0	-14	-12	-2	+7
(6.3.2.1 ²)	+156	-84	-68	-6	-48	-76	+54	+51	+42	+63	+32	+84	+66	-15	-38	-36	-18	+24	-12	+1
(6.2 ³ .1)	+52	-40	+14	-22	-16	-12	+4	+7	+16	+26	+4	+32	+16	-4	-26	-6	+2	+2	+18	-3
(5 ² .1 ³)	+26	-8	-15	-21	-26	+14	+16	+8	+23	+29	+15	-6	-4	-13	-1	0	-7	-11	+5	-4
(5.4.2.1 ²)	+156	-84	-68	-96	-12	+44	+12	+51	+132	+63	+14	-40	-36	-23	-4	+12	-12	+6	+12	+1
(8.1 ⁵)	-13	+1	+2	+3	+4	+5	+13	-1	-3	-4	-2	-5	-5	-6	-6	-3	-7	-7	-7	0
(7.2.1 ⁴)	-65	+17	+10	+15	+20	+25	+23	-6	-17	-23	-10	-29	-25	-35	-30	-15	+1	+7	+7	0
(6.3.1 ⁴)	-65	+17	+32	+15	+20	+25	-19	-17	-19	-23	-14	-29	-31	+7	+4	+6	+7	-3	+7	0
(7.1 ⁶)	+13	-1	-2	-3	-4	-5	-6	+1	+3	+4	+2	+5	+5	+6	+6	+3	0	0	0	0
(5.3 ² .1 ²)	+78	-42	-67	+12	-6	+22	-15	+42	+19	+12	+31	-20	-21	-8	+10	-12	+15	-3	+3	+8
(4 ² .3.2)	-39	+39	+17	+9	-33	+19	-3	-28	-26	+24	+19	+14	-30	-16	+2	+12	+3	+6	-6	-4
(6.2 ² .1 ³)	-130	+58	+9	+30	+40	+50	-17	-14	-37	-52	-18	-66	-47	+4	+28	+12	-1	-2	-7	0
(6.2.1 ⁵)	+78	-18	-12	-18	-24	-30	+6	+7	+20	+27	+12	+34	+30	-1	-6	-3	0	0	0	0
(6.1 ⁷)	-13	+1	+2	+3	+4	+5	0	-1	-3	-4	-2	-5	-5	0	0	0	0	0	0	0
(5.3 ² .2.1)	+156	-120	-24	-6	-84	+84	-30	+54	+34	+81	+24	-20	-66	-20	+38	+6	+12	-6	0	-9
(5.2 ⁴)	+13	-13	+9	-13	+5	-3	+1	+2	+4	+8	0	-2	-6	+2	0	+6	-1	0	-6	+3
(5.4.1 ⁴)	-65	+17	+32	+45	+20	-15	-19	-17	-49	-26	-14	+11	+3	+12	+4	-3	+7	+7	-5	0
(5.3.2.1 ³)	-260	+116	+106	+30	+80	-60	+8	-72	-72	-101	-52	+28	+48	+21	-18	+12	-8	-4	-2	0
(5.2 ³ .1 ²)	-130	+82	-13	+40	+40	-30	+11	-16	-39	-59	-14	+6	+37	-3	-6	-12	+1	+2	+3	0
(5.3.1 ⁵)	+78	-18	-34	-18	-24	+10	+6	+18	+22	+27	+16	-6	-4	-6	+2	-3	0	0	0	0
(5.2 ² .1 ⁴)	+195	-75	-19	-45	-60	+25	-6	+20	+54	+75	+28	-5	-28	+1	+2	+3	0	0	0	0
(5.2.1 ⁶)	-91	+19	+14	+21	+28	-5	0	-8	-23	-31	-14	+1	+5	0	0	0	0	0	0	0
(5.1 ⁸)	+13	-1	-2	-3	-4	0	0	+1	+3	+4	+2	0	0	0	0	0	0	0	0	0
(4.3 ³)	-13	+13	+13	-17	+1	+13	-8	-13	+4	+16	-7	-14	+4	-5	+7	-4	+8	-5	+1	+2
(4 ² .3.1 ²)	+78	-42	-67	-33	+66	+22	-36	+42	+64	-42	-5	-44	+12	+13	-13	+12	+18	+3	-3	-7
(4 ² .2 ² .1)	+78	-60	-12	-48	+48	-18	+6	+27	+62	-18	-33	-8	+24	-6	-8	+6	+3	+6	-12	+3

	(5.4. ²)	(10.1 ³)	(9.2.1 ²)	(8.3.1 ²)	(8.2.1)	(7.4.1 ²)	(7.3.2.1)	(7.2 ³)	(6.5.1 ²)	(6.4.2.1)	(6.3 ² .1)	(6.3.2 ²)	(5 ² .2.1)	(5.4.3.1)	(5.4.2 ²)	(5.3 ² .2)	(4 ³ .1)	(9.1 ⁴)	(8.2.1 ³)	(7.3.1 ³)
(5.4.3.1)	+10	+34	+45	-43	-8	-17	-10	+10	+17	+2	+18	-6	+8	-10	-10	+3	-2	-16	+9	+16
(5.4.2 ²)	-5	+17	+15	+1	-20	-13	+24	0	+1	-12	0	-2	+11	-10	0	+2	+5	-8	+8	-4
(5.3 ² .2)	+1	+8	+24	-10	-19	+5	-1	+10	-1	+17	-3	-10	-11	+3	+2	+4	-1	-8	+8	+2
(4 ³ .1)	-7	+9	-5	-9	+5	-9	+14	-5	+11	-6	-1	+1	-5	-2	+5	-1	+3	0	0	0
(9.1 ⁴)	+4	-1	-4	-5	-5	-6	-12	-2	-15	-14	-7	-7	-16	-16	-8	-8	0	+1	+5	+6
(8.2.1 ³)	-4	-5	-19	-24	-22	-29	-53	-8	-33	-62	-31	-28	+9	+9	+8	+8	0	+5	+24	+29
(7.3.1 ³)	-4	-6	-23	-27	-28	-33	-61	-12	+5	+12	+11	+4	+4	+16	-4	+2	0	+6	+29	+33
(7.2 ² .1 ²)	+4	-9	-32	-42	-33	-51	-83	-10	+2	+29	+12	+26	-3	-9	-2	-5	0	+9	+41	+51
(6.4.1 ³)	-4	-16	-26	-30	-34	+9	+5	+2	+17	+6	+2	-2	+1	+1	+8	-7	0	+7	+34	-3
(6.3.2.1 ²)	+8	-21	-75	-90	-82	+12	+74	+10	0	+2	+21	-14	-8	-10	-4	+5	0	+21	+96	-15
(6.2 ³ .1)	-4	-7	-23	-33	-20	+3	+26	+10	-1	-6	-12	-6	+3	+4	+2	0	0	+7	+30	-2
(5 ² .1 ³)	+6	-8	-31	+3	+2	+5	+1	-1	+11	+2	-5	+1	+8	-8	-4	+4	0	+8	-1	-2
(5.4.2.1 ²)	-8	-51	-75	+22	+22	+24	-12	0	+8	+2	-15	+4	-10	+10	0	-1	0	+24	-10	-6
(8.1 ⁵)	0	+1	+4	+5	+5	+6	+12	+2	+7	+14	+7	+7	0	0	0	0	0	-1	-5	-6
(7.2.1 ⁴)	0	+6	+23	+29	+27	+35	+65	+10	-1	-8	-4	-7	0	0	0	0	0	-6	-29	-35
(6.3.1 ⁴)	0	+7	+27	+32	+33	-3	-11	0	-2	-4	-7	+3	0	0	0	0	0	-7	-34	+3
(7.1 ⁶)	0	-1	-4	-5	-5	-6	-12	-2	0	0	0	0	0	0	0	0	0	+1	+5	+6
(5.3 ² .1 ²)	-4	-12	-43	+14	+10	+6	+18	-10	-7	-6	0	+2	-4	0	+4	-2	0	+12	-5	-9
(4 ² .3.2)	+3	+18	-18	-4	+11	0	+4	-5	+2	-2	-6	+5	-2	+8	-1	-2	-3	0	0	0
(6.2 ² .1 ³)	0	+14	+51	+66	+55	-4	-32	-10	+1	+3	+4	+2	0	0	0	0	0	-14	-65	+4
(6.2.1 ⁵)	0	-7	-27	-34	-32	+1	+7	+2	0	0	0	0	0	0	0	0	0	+7	+34	-1
(6.1 ⁷)	0	-1	+4	+5	+5	0	0	0	0	0	0	0	0	0	0	0	0	-1	-5	0
(5.3.2 ² .1)	+4	-24	-79	+17	+46	-6	-7	-10	+2	-16	+3	+6	+11	-1	-2	0	0	+24	-17	0
(5.2 ⁴)	0	-2	-6	0	+6	0	-6	0	0	+6	0	0	-3	0	0	0	0	+2	-2	+2
(5.4.1 ⁴)	0	+17	+30	-5	-1	-8	+2	0	-7	-2	+5	-1	0	0	0	0	0	-8	+1	+2
(5.3.2.1 ³)	0	+32	+117	-19	-28	-5	-15	+10	+3	+8	-1	-2	0	0	0	0	0	-32	+11	+8
(5.2 ³ .1 ²)	0	+16	+55	-5	-27	+3	+12	0	-1	-3	0	0	0	0	0	0	0	-16	+9	-4
(5.3.1 ⁵)	0	-8	-31	+3	+2	+2	+4	-2	0	0	0	0	0	0	0	0	0	+8	-1	-2
(5.2 ² .1 ⁴)	0	-20	-74	+5	+18	-1	-3	0	0	0	0	0	0	0	0	0	0	+20	-6	+1
(5.2.1 ⁶)	0	+8	+31	-1	-3	0	0	0	0	0	0	0	0	0	0	0	0	-8	+1	
(5.1 ⁸)	0	-1	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1		
(4.3 ³)	-1	+3	-3	-3	+3	+6	-3	0	-3	-3	+3	0	+3	-3	0	0	+1			
(4 ² .3.1 ²)	+4	-27	+11	+20	-11	+9	-13	+5	-13	+5	0	-1	+5	-1	-2	+1				
(4 ² .2 ² .1)	+2	-27	+19	-1	-5	+9	-12	+5	-3	+2	+6	-3	-1	-2	+1					

